Trig Summary

Angles

Acute: $0° < \theta < 90°$

Right: $\theta = 90°$

Obtuse: $90° < \theta < 180°$

Straight: $\theta = 180°$

A and B are **complementary** if $A + B = 90°$.

A and B are **supplementary** if $A + B = 180°$.

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

The Area of a Triangle

If a and b are sides and C is the included angle:

$$A = \frac{1}{2}ab\sin C$$

Heron's Formula

If a, b, and c are the sides of a triangle and if s is defined by

$$s = \frac{1}{2}(a + b + c)$$

Then the area is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Angles in a Triangle

The **sum of the interior angles** of any plane (flat) triangle is 180°.

The Pythagorean Theorem

If a and b are the legs of a right triangle (one with a 90° angle in it) and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

The 45°-45°-90° Triangle

The two legs are equal, since it's an isosceles triangle.

The 30°-60°-90° Triangle

The side opposite the 30° angle is half the hypotenuse – OR – the longest side of the triangle is twice the shortest side.

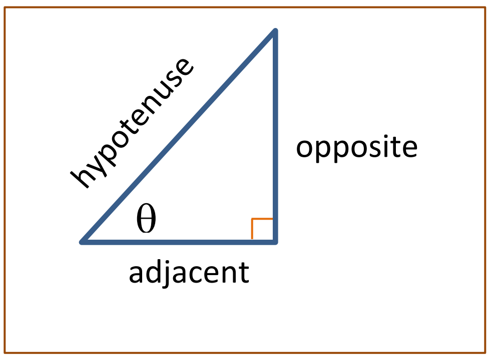
The Triangle Inequality

The sum of any two sides of a triangle must exceed the third side.

Sides of 4, 5, and 7 make a triangle, since any two sides added together is larger than the third side.

Sides of 5, 7, and 13 do NOT make a triangle, since 5 + 7 is not larger than 13.

Right Triangle Trig



$$\text{sine}(\theta) = \mathbf{sin}\,\boldsymbol{\theta} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosine}(\theta) = \mathbf{cos}\,\boldsymbol{\theta} = \frac{\text{adj}}{\text{hyp}}$$

$$\text{tangent}(\theta) = \mathbf{tan}\,\boldsymbol{\theta} = \frac{\text{opp}}{\text{adj}}$$

$$\text{cotangent}(\theta) = \mathbf{cot}\,\boldsymbol{\theta} = \frac{\text{adj}}{\text{opp}}$$

$$\text{secant}(\theta) = \mathbf{sec}\,\boldsymbol{\theta} = \frac{\text{hyp}}{\text{adj}}$$

$$\text{cosecant}(\theta) = \mathbf{csc}\,\boldsymbol{\theta} = \frac{\text{hyp}}{\text{opp}}$$

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
cot	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$
sec	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2
csc	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$

The Big 8 Identities

The Reciprocal Identities

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

The Ratio Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

The Cofunction Identities

$$\sin\theta = \cos(90° - \theta)$$
$$\cos\theta = \sin(90° - \theta)$$
$$\tan\theta = \cot(90° - \theta)$$
$$\cot\theta = \tan(90° - \theta)$$
$$\sec\theta = \csc(90° - \theta)$$
$$\csc\theta = \sec(90° - \theta)$$

Note:
θ and 90° – θ are complementary angles, since their sum is 90°:

$$\theta + (90° - \theta)$$
$$= \theta + 90° - \theta$$
$$= 90°$$ ✓

The Even/Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\cot(-\theta) = -\cot\theta$$
$$\sec(-\theta) = \sec\theta$$
$$\csc(-\theta) = -\csc\theta$$

The functions *cos* and *sec* are called "**even** functions"; the other four are called **odd** functions.

Radians and Degrees

Fact: **π radians = 180°**

To convert degrees to radians, multiply the degrees by $\frac{\pi}{180}$.

To convert radians to degrees, multiply the radians by $\frac{180}{\pi}$.

Trig in the Plane

1. Sketch θ in standard position.
2. Find any point (not the origin) on the terminal ray of θ. This gives you values for x and y.
3. Calculate r, the distance from the origin to the point (x, y):

$$r = \sqrt{x^2 + y^2}$$

4. This gives you all the data you need to calculate any of the six trig functions:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$
$$\sec\theta = \frac{r}{x} \qquad \csc\theta = \frac{r}{y}$$

	Q I	Q II	Q III	Q IV
sin	Pos	Pos	Neg	Neg
cos	Pos	Neg	Neg	Pos
tan	Pos	Neg	Pos	Neg
cot	Pos	Neg	Pos	Neg
sec	Pos	Neg	Neg	Pos
csc	Pos	Pos	Neg	Neg

Degrees	Radians	***sin***	***cos***	***tan***	***cot***	***sec***	***csc***
0°	0	0	1	0	Und	1	Und
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	Und	0	Und	1
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
180°	π	0	-1	0	Und	-1	Und
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$
270°	$\frac{3\pi}{2}$	-1	0	Und	0	Und	-1
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
315°	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2
360°	2π	0	1	0	Und	1	Und

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \qquad \sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \qquad \cos(A-B) = \cos A\cos B + \sin A\sin B$$

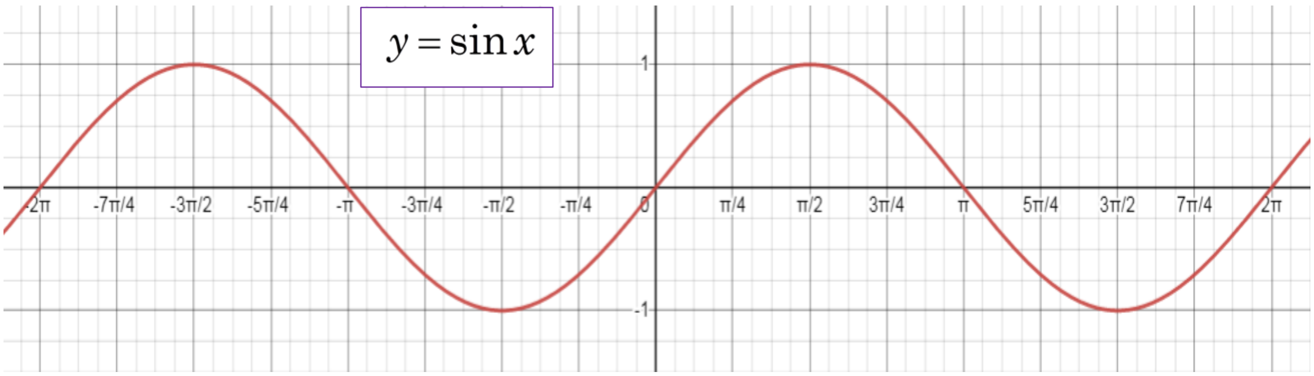
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 1-2\sin^2 A \\ &= 2\cos^2 A - 1\end{aligned}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Inverse trig functions



Domain = $\mathbb{R}$ or, all x's

Range = $[-1, 1]$ or, $-1 \leq y \leq 1$

Amplitude = 1

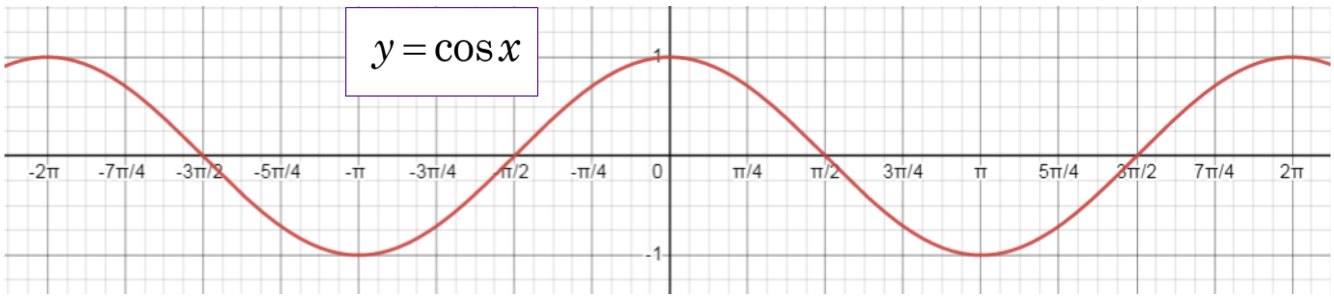
Period = 2π

Amplitude = $|A|$ Period = $\frac{2\pi}{B}$

Horizontal (Phase) Shift:
- If $C > 0$, C units to the left
- If $C < 0$, C units to the right

Vertical Shift:
- If $D > 0$, D units up
- If $D < 0$, D units down

$$y = A\sin\left(B(x+C)\right)+D$$



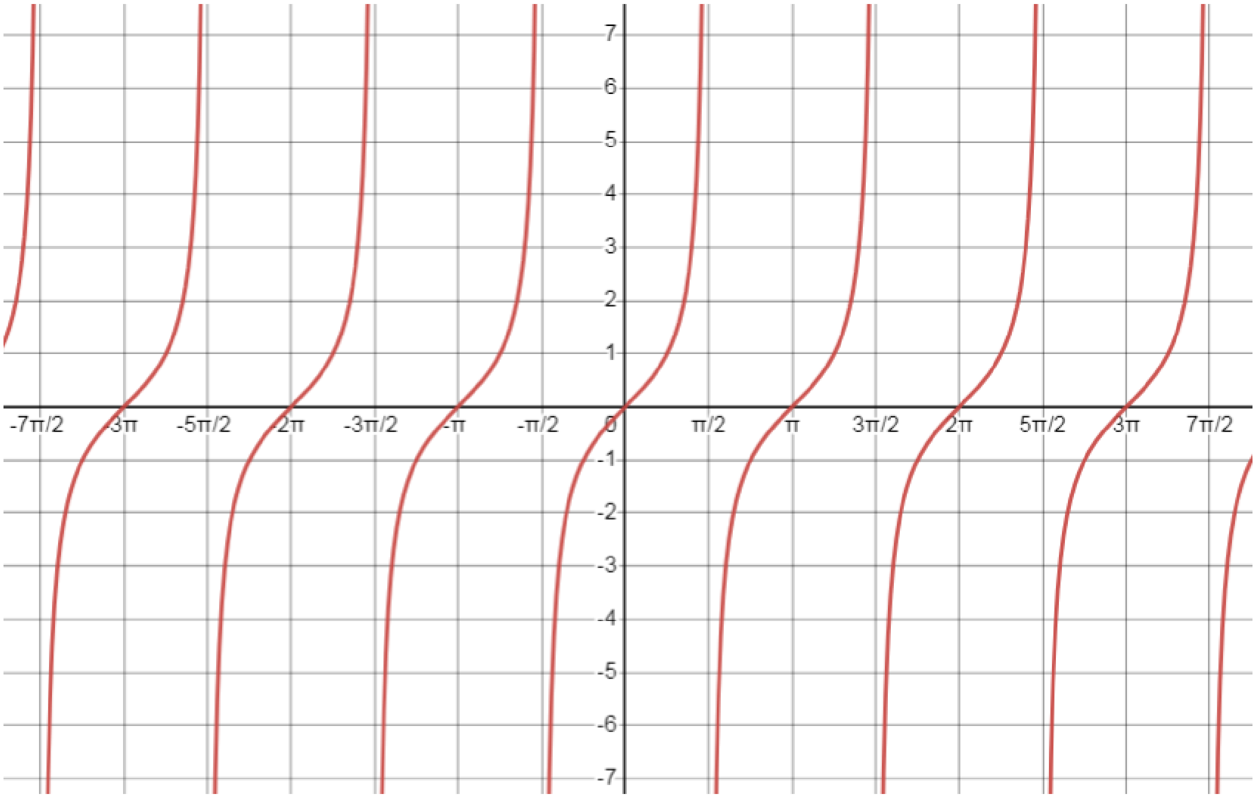
Domain = $\mathbb{R}$ or All x's

Range = $[-1, 1]$ or $-1 \le y \le 1$

Amplitude = 1

Period = 2π

$y = \tan x$



Domain $= $ All angles except $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$

$= $ All angles except ODD multiples of $\frac{\pi}{2}$.

$= \mathbb{R} - \left\{(2k+1)\frac{\pi}{2}, k \text{ an integer}\right\}$

Range = $\mathbb{R}$ or $(-\infty, \infty)$

Amplitude = Undefined

Period = π